INSTRUCTIONAL AND RESEARCH AIDS FOR COLLEGES AND UNIVERSITIES

LOGIC TRAINING AID

"A professor can never better distinguish himself than by encouraging a clever pupil, for the true discoverers are among them, as comets amongst the stars."

CARL LINNAEUS
1707-1778

LOGIC TRAINING AID

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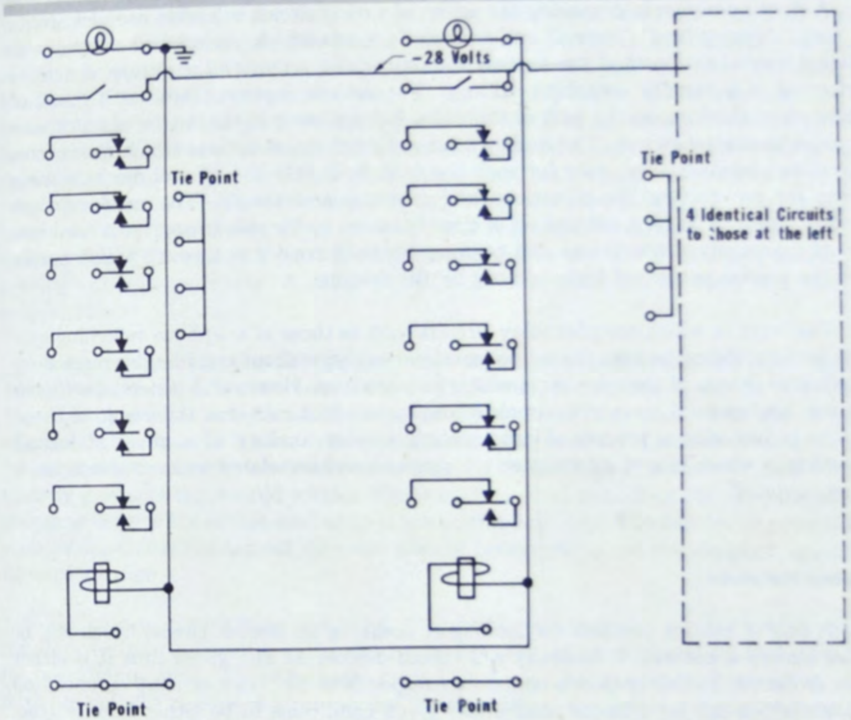
ABOUT YOUR LOGIC TRAINING AID

The Logic Training Aid was developed for laboratory experimentation in logic courses by the Western Electric Graduate Engineering Training Center in Winston-Salem, North Carolina. Because of its significant value in the practical application of mathematically derived circuitry, a limited quantity of the Training Aids have been produced for distribution to colleges and universities.

The information in this booklet is designed to give students a basis in the skills required for developing logic circuits. Circuitry thus developed may be wired into the Logic Training Aid for determination of final results.

LOGIC TRAINING AID



List of Components

Pin Jacks, White	42
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Switch, Toggle, DPST	6
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Wire	200 Ft.
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THE LOGIC OF RELAY CIRCUITS

A circuit designer in discussing the action of a relay circuit will often use such words as "tell," "remember," "know," and "choose"; words which are applied normally to rational mental actions and yet express with surprising accuracy the nature of actions performed in automatic switching systems. The actions, however, are not a result of independent thinking on the part of the units, but are more in the nature of automatic response to control signals. The designer has analyzed the situations which may occur, determined what shall be done for each one, and built into the control mechanisms a means for recognizing the situations and reacting accordingly. The machine then "knows" what to do for a definite set of circumstances and is able to analyze a combination of preconceived conditions and produce for each condition a result which agrees with the precise pattern of logic laid out by the designer.

The ways in which complex relay circuits such as those of telephone switching systems perform logical actions cannot be explained easily, without considerable discussion of technical details of the systems in which they are used. However, a general discussion of a few fundamental ideas with simple examples will illustrate that the design of these circuits is basically a process of constructing a relay analogy of a chain of logical reasoning in which bits of information are received and correlated and suitable actions taken.

Contact Networks

A pair of mating contacts for making or breaking an electric circuit which will be called simply a contact, is basically a 2-valued device. At any given time it is either open or closed. In this respect a contact corresponds to an "open or shut" proposition in logic which can be demonstrated under given conditions to be either false or true. A 2-terminal network of contacts is similar to a single contact in that it is either open or closed for any given set of conditions. By 2-terminal network is meant a network consisting of individual contacts connected together in a configuration which provides but two entry points. Such networks connected in a circuit with a battery can control the actions of relays in a complex control circuit. A 2-terminal network corresponds to a proposition whose truth or falsity depends on the truth or falsity of a number of individual propositions, the individual propositions corresponding to the individual contacts in the network. The development of a contact network to perform a certain function is an exercise in logic of the kind which is used in building up a proof that a certain statement is true or false by demonstrating that a number of other statements are true or false. Furthermore, the arrangement of the individual contacts in the network is similar to the way the various statements are related to prove that the given statement is true or false.

Two-terminal contact networks correspond to propositions which may sometimes be true and at other times false and not to propositions which are basically true. (These would prove to be short circuits.) For example, the proposition that "the 5:30 train will arrive on time" may be either true or false and will depend on the truth of a number of other propositions such as (a) "it left the last station on time," and (b) "the track is blocked." The original proposition may be true if proposition (a) is true and (b) is false. The use of a false proposition to prove the truth of another corresponds to the use of break contacts on a relay which close a circuit path only when the relay is not operated. A relay may control two types of contacts, both makes which close when the relay operates and breaks which open when the relay operates. A complete relay may be compared to a proposition which is sometimes true (operated) and sometimes false (released) and the contacts on this relay correspond to the use of this proposition in proving other propositions. A relay circuit thus is a collection of open and closed propositions.

Requirements for relay circuits are usually stated on a cause and effect basis; that is when "such and such" events happen in a certain way, certain effects will be produced. If a clear logical statement of the relations between certain causes and the desired effects can be made, it is usually a simple matter to construct a circuit of relays controlled by contact networks which is analogous to the logical statement and consequently produces the desired results. Flaws in the logical reasoning will produce corresponding flaws in the circuit leading to inoperative conditions. Also restrictive conditions such as apparatus limitations may not always permit us to use the simplest straightforward solution.

The causes, constituting the input or control conditions for a relay circuit must result, directly or indirectly, in the closure (or opening) of an electrical contact suitable for operating a relay. Thus the control conditions may be: the manual operation of a push button; the closure of a contact by the mechanical motion of some instrument or

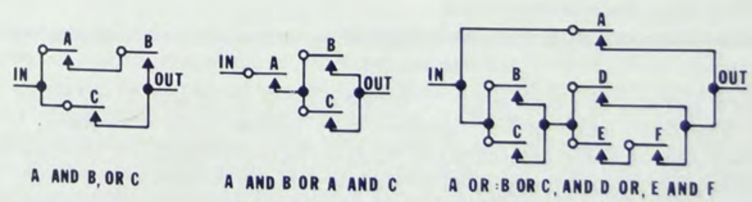


Figure 1. Circuit networks as statements of conditions for closing a path.

machine; the insertion of a plug into a jack; or the closure of relay contacts in other associated circuits.

If the purpose of a relay circuit is merely to recognize the simultaneous occurrence of several control conditions and produce some output condition when this combination of conditions occurs, the correspondence between a statement of conditions and a relay contact network is very close. For example, if a lamp is to be lighted when two push button contacts, A and B, are both operated at the same time the statement of conditions for lighting the lamp is "A is operated *and* B is operated." The circuit path obviously is a series connection of make contacts. On the other hand, if the conditions are that the lamp shall light if "A is operated *or* B is operated," the circuit consists of a parallel connection of make contacts A and B. In the foregoing statements the words "and" and "or" have been italicized. These conjunctions in a careful statement of conditions for closing a circuit indicate whether a paralleled or series arrangement of the contacts is to be used, *and* indicating a series connection while *or* indicates a paralleled connection.

The same process may be applied to more elaborate statements. The path which is closed when "A and B are operated, or C is operated" is the series parallel arrangement of Figure 1(A) while the circuit which is closed when "A is operated and B or C is operated" is shown in Figure 1(B). A still more elaborate example is shown in Figure

A AND NOT-B, OR, NOT-A AND B

Figure 2. Use of break contacts.

1(C) where the path is closed when "A is closed or: B or C, and; D or, E and F are closed." The punctuation of the statement may become quite complicated but the above examples show that an accurate analysis of the requirements for closing a circuit will indicate directly a series parallel arrangement of the contact network. The point is not that a verbal statement of conditions for closing a circuit path is necessary or desirable but that the mental processes required in constructing and punctuating such a statement are the same as those required in designing the network. Note that the or indicating a parallel connection is not a mutually exclusive term. A parallel connection of two contacts A and B is a circuit which is closed when "A or B or both are closed."

It is important that statements of circuit conditions mean precisely what is intended For example, consider a statement describing the control of circuit paths X and Y by contacts on relays A and B, "When A operates, path X is closed but when both A and B operate only path Y is closed." This does not specifically state the conditions for closing path X. What is meant is that X closes when "A is operated and B is not oper-

A AND NOT-B, OR, A AND C
(A)

A AND NOT-B, OR, A AND C
(B)

Figure 3. Simplification by restatement.

ated." When it is necessary to state that a relay is not operated or some condition is not present in order to describe a control condition the use of a break contact is implied. Another example is a statement such as "the path is closed when either A or B is operated" where the or is mutually exclusive and the circuit is open when both A and B are operated. A more exact statement of these conditions is, "A operated and B not operated, or A not operated and B operated." The circuit is shown in Figure 2 where transfer contact arrangements are used.

After a little study it becomes evident that the sketch of a network of contacts corresponds to a concise statement of conditions for opening or closing a circuit path. The most concise statement of a set of conditions or, in other words, the contact network producing the desired results with the least number of contacts, is often reached only after several trials and restatements of the conditions. As a simple example, the circuit which is closed when "A is operated and B is not operated, or, A is operated and C is operated," as shown in Figure 3(A), may be restated as a circuit which is closed when "A is operated and, B is not operated, or C is operated." Since the element A is mentioned only once, a single A contact is sufficient as shown in Figure 3(B).

The practicing circuit designer seldom makes actual verbal or written statements of circuit conditions, but sketches the circuits themselves as the most direct way of expressing the requirements or conditions of a circuit. Often the original circuit contains many superfluous contacts, the important point at this stage being to insure that the various contact networks are closed for all required conditions and not closed for any conditions likely to produce undesirable reactions. Next comes the process of simplification, an elementary example of which was shown in Figures 3(A) and 3(B). All of this requires considerable attention to detail and logical analysis. The designer must keep in mind the positions of all relays of the circuit at any given time, and must visualize the reactions which take place when a contact network closes to operate a relay whose contacts in turn open or close to cause other relays to act, causing still further reactions. An experienced designer often conceives his original circuit network in their simplified forms. After a certain amount of practice, intuition for the construction and simplification of contact networks develops from the repeated application of logic to problems of cause and effect.

The algebra of logic, or Boolean algebra, has been applied to problems of contact network configurations. This is a 2-valued algebra which deals with propositions in logic which are either true or false. Since a contact also is 2-valued, the theorems of this algebra concerning the combination of several propositions into a single proposition also apply to the combination of contacts into a single 2-terminal network. When a network which meets requirements has been devised from a study of the conditions of a problem, this algebra can often be used to great advantage in simplifying the network and eliminating superfluous elements.

A different type of reasoning often leads more directly to the desired form of a contact network. This is, thinking in terms of open circuits or considering the conditions which must open a circuit path, rather than those which close it, and designing a network to satisfy these open circuit conditions. This can be characterized as negative reasoning. For example, consider a network on the contacts of two relays, A and B,

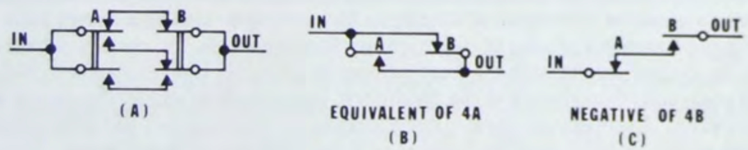


Figure 4. Negative reasoning.

which must be closed when "A and B are released, or, A is operated and B is released, or, A and B are operated." The network corresponding to this statement is shown in Figure 4(A). However, an analysis of the conditions shows that the path is closed for three of the four possible combinations in which A and B may stand operated or released, and is open only when A is released and B is operated. This analysis gives us the equivalent circuit shown in Figure 4(B) which is open only on this condition and consists of a make contact on A in parallel with a break on B.

Now consider the condition "A released and B operated." The circuit closed for this condition, a break on A in series with a make on B is shown in Figure 4(C). This circuit, 4(C), is open when that of Figure 4(B) is closed and closed when 4(B) is open. The two circuits are said to be "negatives" of each other. Either one or the other is closed for any given set of conditions but they are never both closed at the same time. Comparing the circuits of Figure 4(B) and (C), we see that where one contains make contacts the other contains break contacts and a series connection in one becomes a parallel connection in the other. It turns out to be a general rule that the "negative" of any series parallel 2-terminal network of contacts can be constructed by substituting break contacts for

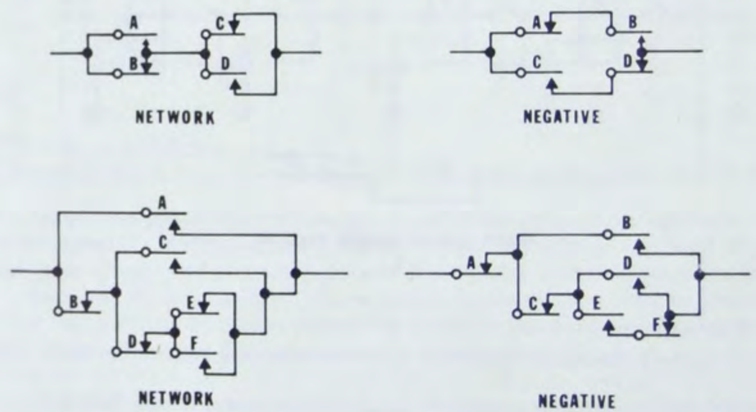
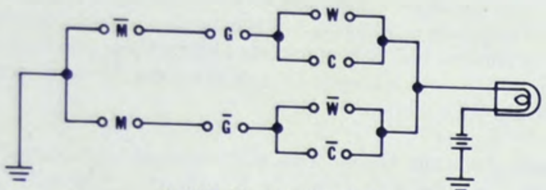
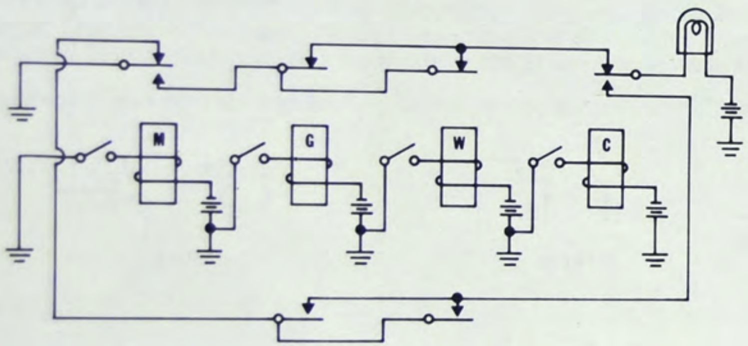


Figure 5. Examples of networks and their negatives.

make contacts and vice versa, and interchanging series and parallel connections. Two examples are shown in Figure 5. Obviously a circuit can be constructed directly from a statement of conditions for opening a path by changing meanings of and and or to



ALARM CIRCUIT NETWORK
(A)



ALARM CIRCUIT WIRING DIAGRAM
(B)

Figure 6. Alarm circuit.

often leads more directly to simplified circuit arrangements. It may be used as a check on circuit arrangements obtained by direct reasoning and the ability to readily construct the negative of any given circuit is a convenient tool in developing relay control circuits.

Example 1

Now let us consider a simple problem in logic to illustrate how a contact network analogy may be constructed, choosing a problem which is obviously one of logic and not electrical circuits. In this it will be seen that in order to develop the circuit the logic must be completely reasoned through. When this logic has been incorporated in the circuit, the circuit then "knows" the correct answers and will deliver them without error. The problem is a simplified version of the well known "wolf, goat, and cabbage" puzzle:

A farmer has entrusted the care of a goat, a wolf, and a quantity of cabbages to a not-too-intelligent hired man. These items may be kept in either of two barns. If the goat is left alone with the cabbages it will eat the cabbages, while the goat himself is subject to being devoured by the wolf if the hired man is not present to intervene. The farmer desires a device to check on the hired man which is to consist of keys W, G, C, and M, representing the wolf, goat, cabbages, and hired man, respectively. By observing the contents of one of the barns and operating the corresponding keys, the farmer is to be warned by the lighting of a red lamp if the hired man is not properly fulfilling his duties.

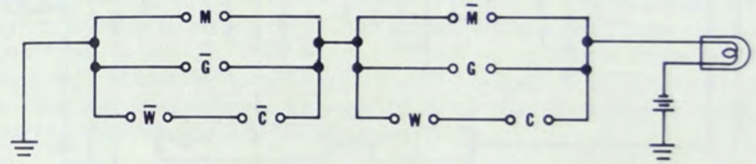


Figure 7. Check circuit.

For a logical analysis of the problem, first consider the situation in a particular barn under observation. If the hired man is not present and the goat is present with the wolf or the cabbages, trouble will occur in that barn. A more concise statement is "M absent and G present and, W or C present." Items absent from one barn must be present in the other so that a similar statement concerning items not observed describes conditions for trouble in the other barn. The statement is "M present and G absent and, W or C absent." Networks can be constructed from these statements, "present" indicating a make contact and "absent" a break. Since trouble will occur if one or the other of these statements is true the two networks must be placed in parallel. This is shown in Figure 6(A) where a bar over a designation indicates a break contact. The equivalent schematic is shown in Figure 6(B). With this device it is necessary only that a person be able to identify the items in one of the barns. No analysis or even knowledge of the habits of wolves or goats is required. The contact network having been developed to correspond to a logical framework will examine input conditions to produce answers according to this logic.

Now suppose that the farmer, after using the foregoing device for some time and losing a quantity of cabbages due to a burned-out warning lamp, requests that a green lamp also be provided to indicate as a check that conditions are satisfactory. Thus, the red lamp will light when the hired man is in trouble, and the green lamp when the situation is under control. For this circuit we may simply construct the negative of Figure 6(A) without further analysis of the problem. The resulting network is shown in Figure 7(A) which is the negative of Figure 6(A). An experienced designer will combine and rearrange the complete circuit for both lamps in the equivalent form shown in Figure 8. We leave to the reader the design of a suitable carrying case for the device.

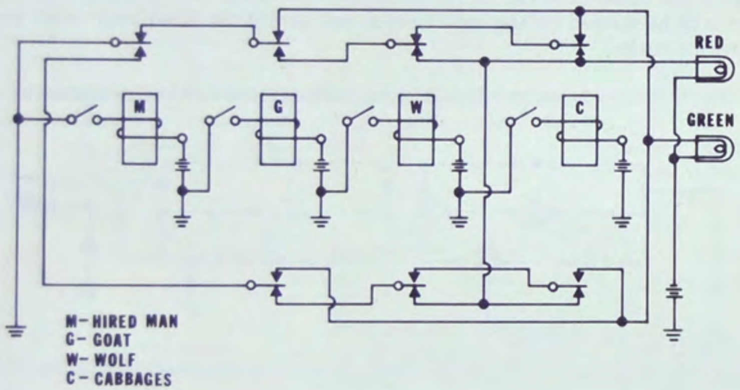
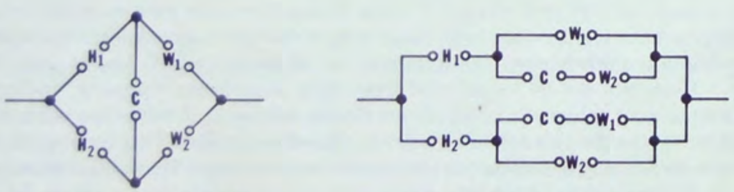


Figure 8. Complete circuit combining alarm and check actions.



A SIMPLE BRIDGE CIRCUIT (A)

SERIES - PARALLEL EQUIVALENT (B)

Figure 9. Bridge circuits.

Further Aspects of Contact Networks

The human mind seems to function most often in an "and-or," or series-parallel type reasoning, and all networks developed directly by processes described before are of the series-parallel type. By using bridge-type configurations, a contact network may describe a set of conditions much more concisely than it may be stated in words or even thought about. This can be shown by the example of the two jealous wives whose husbands may not take a canoe ride with the other wife except if accompanied by a chaperon. The conditions for a couple to take a canoe trip are concisely expressed by paths through the bridge network of Figure 9(A) where W1 and W2 represent the two wives, H1 and H2 their respective husbands and C designates the chaperon.

A bridge-type network is one which has one or more cross branches between parallel branches. Figure 9(A) is an example of the simplest bridge, and a series-parallel equivalent is shown in Figure 9(B) where it is necessary to duplicate a number of the contacts. The conditions for closing this bridge network consisting of five contacts cannot be described adequately by mentioning each contact only once. A circuit network developed to satisfy given requirements originates usually as a series parallel arrangement. There seems to be no general method for converting a given series-parallel network to a bridge network, or even determining that an equivalent bridge network is possible. Bridge networks with their resulting saving in contacts are developed usually by inspection and trial, followed by a careful check that the bridge is opened and closed under all the conditions of the original problem.

A given number of relays or switches ,n, may stand open or closed in 2n possible combinations. That is, a single relay may be either operated or released, two relays, A and B may stand in four combinations, namely, both released, A operated, B operated, or both operated. Three relays give eight combinations and each additional relay doubles the number of combinations. A circuit path can be developed for each of these combinations which is closed only when this combination occurs and open during all others. In fact, a network can be developed which is closed for any set of combinations and open for all others. In the problem of the farmer there are 16 ways in which the wolf, goat, cabbages, and man can be placed in and out of the barn and the circuit for the red lamp is closed for six of these combinations. A routine method for constructing a network closed for any set of combinations of given relays is to first construct the path for each individual combination. Each of these consists of a series chain of one contact on each relay, the contact being a make or a break, depending on whether the relay it operated or released in that combination. The final network may ne obtained by placing all of the individual chains in parallel and simplifying the resulting network.

A surprisingly large number of distinctly different paths may be constructed on the contacts of a few relays. Consider the case of two relays which may stand in for different

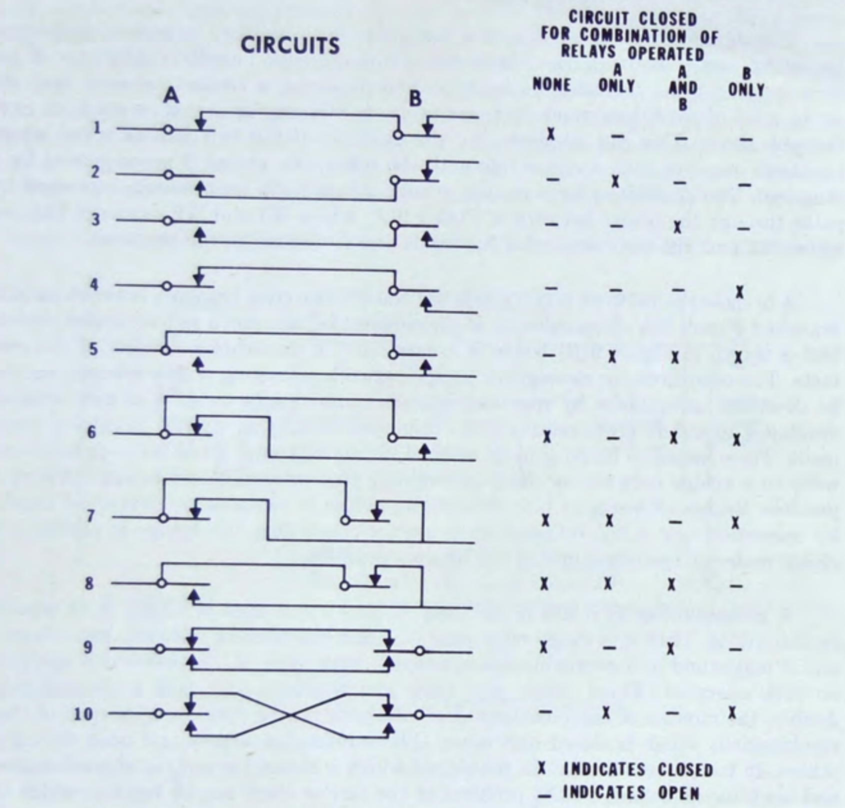


Figure 10. The ten different circuits on the contacts of two relays.

combinations. The four combinations may be designated as either open or closed in 16 ways, and 16 corresponding circuits developed. Of these, one is an open circuit, one is a closed circuit, four require contacts on only one relay, and the remaining ten require contacts on both relays and are all different. These are shown in Figure 10. For three relays the number of distinctly different arrangements is 218, which indicate how rapidly the complexity of a relay circuit arrangement may increase as the number of relays increases.

THE BOOLEAN ALGEBRA OF 0 AND 1

The Boolean Algebra of 0 and 1 is a mathematical system composed of these two elements. These two elements are not the ordinary 0 and 1 of our number system, but, in many respects, they behave just the same. In this system, there will be two operations: addition and multiplication. Hence, one knows how to add and multiply the two elements. The variables will be denoted by X, Y, Z, W, etc. One must remember that this system is quite simple because these variables can assume only the values 0 and 1.

Our postulates, or assumptions, are:

1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$
3. $1 \cdot 1 = 1$
4. $0 + 0 = 0$
5. $1 \cdot 0 = 0.1 = 0$
6. $0 + 1 = 1 + 0 = 1$

Only one of the above postulates would give anyone any trouble in our everyday arithmetic. Please note that no interpretation has been placed on the two elements. We will do this in the application of our system later on. We will now prove some theorems without any interpretation on our elements or operations so that we may use these theorems in any situation in which our postulates hold true. The proofs will be by exhaustion in most cases.

Theorem 1a: $X + Y = Y + X$

Theorem 1b: $X \cdot Y = Y \cdot X$

Proof of Theorem 1.

X	Y	X + Y	Y + X	X · Y	Y · X
0	0	0	0	0	0
0	1	1	1	0	0
1	0	1	1	0	0
1	1	1	1	1	1

Theorem 2a: (X+Y) + Z = X + (Y+Z)

Theorem 2b: (X · Y) · Z = X · (Y · Z)

Proof of Theorem 2.

X	Y	Z	(X+Y)	(Y+Z)	(X+Y)+Z	X+(Y+Z)	X · Y	Y · Z	(X · Y · Z)	X · (Y · Z)
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	1	1	0	0	0	0
0	1	0	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	0	1	0	0
1	0	0	1	0	1	1	0	0	0	0
1	0	1	1	1	1	1	0	0	0	0
1	1	0	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	1	1	1	1

Theorem 3a: X · Y + X · Z = X · (Y+Z)

Theorem 3b: (X+Y) · (X+Z) = X+Y · Z

Proof of Theorem 3.

X	Y	Z	X · Y	X · Z	Y+Z	X · Y+X · Z	X(Y+Z)	X+Y	X+Z	Y · Z	(X+Y)·(X+Z)	X+Y · Z
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	0	0
0	1	1	0	0	1	0	0	1	1	1	1	1
1	0	0	0	0	0	0	0	1	1	0	1	1
1	0	1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	0	1	1	1	1	1	0	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1

Theorem 4a: X + X = X

Theorem 4b: X · X = X

Proof of Theorem 4.

X	X+X	X · X
0	0	0
1	1	1

Theorem 5a: X + X · Y = X.

Theorem 5b: X · (X + Y) = X.

Proof of Theorem 5.

X	Y	X · Y	X + X · Y	X + Y	X · (X + Y)
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	1	1
1	1	1	1	1	1

A new concept will be introduced now called $\overline{X}$. $\overline{X}$ means not X. Hence, if X has the value 1, then $\overline{X}$ has the value 0, or, if X has the value 0, then $\overline{X}$ has the value 1.

Theorem 6: $(\overline{\overline{X}}) = X.$

Proof of Theorem 6.

X	$\overline{X}$	$(\overline{\overline{X}})$
0	1	0
1	0	1

Theorem 7a: $\overline{(X + Y)} = \overline{X} \cdot \overline{Y}$

Theorem 7b: $\overline{(X \cdot Y)} = \overline{X} + \overline{Y}$

Proof of Theorem 7.

X	Y	$X + Y$	$\overline{X}$	$\overline{Y}$	$\overline{(X + Y)}$	$\overline{X} \cdot \overline{Y}$	$X \cdot Y$	$\overline{(X \cdot Y)}$	$\overline{X} + \overline{Y}$
0	0	0	1	1	1	1	0	1	1
0	1	1	1	0	0	0	0	1	1
1	0	1	0	1	0	0	0	1	1
1	1	1	0	0	0	0	1	0	0

Theorem 8a: $X + \overline{X} = 1$

Theorem 8b: $X \cdot \overline{X} = 0$

Proof of Theorem 8.

X	$\overline{X}$	$X \cdot \overline{X}$	$X + \overline{X}$
0	1	0	1
1	0	0	1

Theorem 9a: $0 + X = X$

Theorem 9b: $1 \cdot X = X$

Proof of Theorem 9.

X	$0 + X$	$1 \cdot X$
0	0	0
1	1	1

Theorem 10a: $1 + X = 1$

Theorem 10b: $0 \cdot X = 0$

Proof of Theorem 10.

X	$1 + X$	$0 \cdot X$
0	1	0
1	1	0

Theorem 11a: $X(\overline{X} + Y) = X\ Y$

Theorem 11b: $X + (\overline{X}\ Y) = X + Y$

Proof of Theorem 11.

X	Y	$\overline{X}$	$\overline{X} + Y$	$X(\overline{X} + Y)$	$X \cdot Y$	$\overline{X} \cdot Y$	$X + \overline{X} \cdot Y$	$X + Y$
0	0	1	1	0	0	0	0	0
0	1	1	1	0	0	1	1	1
1	0	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1

The proof of theorem 11 will now be given by using previous theorem. This is the method that one usually follows in mathematics.

Proof of Theorem 11a.

$$
\begin{aligned}
X \cdot (\overline{X} + Y) &= X \cdot \overline{X} + X \cdot Y && \text{by theorem 3a} \\
&= 0 + X \cdot Y && \text{by theorem 8b} \\
&= X \cdot Y && \text{by theorem 9a}
\end{aligned}
$$

Proof of Theorem 11b.

$$
\begin{aligned}
X + (\overline{X} \cdot Y) &= X + Y \cdot \overline{X} && \text{by theorem 1b} \\
&= (X + Y) \cdot (X + \overline{X}) && \text{by theorem 3b} \\
&= (X + Y) \cdot 1 && \text{by theorem 8a} \\
&= (X + Y) && \text{by theorem 1b and 9b}
\end{aligned}
$$

Theorem 12a: $(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)$

Theorem 12b: $X \cdot Y + \overline{X} \cdot Z + Y \cdot Z = X \cdot Y + \overline{X} \cdot Z$

Proof of Theorem 12a.

X	Y	Z	$\overline{X}$	$X + Y$	$\overline{X} + Z$	$Y + Z$	$(X + Y) \cdot (\overline{X} + Z)$	$(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z)$
0	0	0	1	0	1	0	0	0
0	0	1	1	0	1	1	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	0	0	0
1	0	1	0	1	1	1	1	1
1	1	0	0	1	0	1	0	0
1	1	1	0	1	1	1	1	1

Proof of Theorem 12b.

X	Y	Z	$\overline{X}$	$X \cdot Y$	$\overline{X} \cdot Z$	$Y \cdot Z$	$X \cdot Y + \overline{X} \cdot Z$	$X \cdot Y + \overline{X} \cdot Z + Y \cdot Z$
0	0	0	1	0	0	0	0	0
0	0	1	1	0	1	0	1	1
0	1	0	1	0	0	0	0	0
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	1	1	0	1	0	1	1	1

Theorem 13a: $(X + Y) \cdot (\overline{X} + Z) = X \cdot Z + \overline{X} \cdot Y$

Theorem 13b: $(X \cdot Y) + (\overline{X} \cdot Z) = (X + Z) \cdot (\overline{X} + Y)$

Proof of Theorem 13a.

X	Y	Z	$\overline{X}$	$X + Y$	$\overline{X} + Z$	$X \cdot Z$	$\overline{X} \cdot Y$	$(X + Y) \cdot (\overline{X} + Z)$	$X \cdot Z + \overline{X} \cdot Y$
0	0	0	1	0	1	0	0	0	0
0	0	1	1	0	1	0	0	0	0
0	1	0	1	1	1	0	1	1	1
0	1	1	1	1	1	0	1	1	1
1	0	0	0	1	0	0	0	0	0
1	0	1	0	1	1	1	0	1	1
1	1	0	0	1	0	0	0	0	0
1	1	1	0	1	1	1	0	1	1

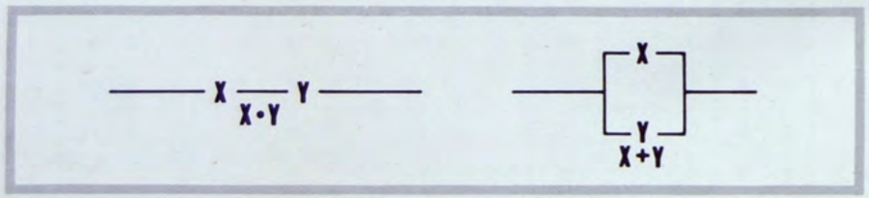
Proof of Theorem 13b.

X	Y	Z	$\overline{X}$	$X \cdot Y$	$\overline{X} \cdot Z$	$X + Z$	$\overline{X} + Y$	$(X \cdot Y) + (\overline{X} \cdot Z)$	$(X + Z) \cdot (\overline{X} + Y)$
0	0	0	1	0	0	0	1	0	0
0	0	1	1	0	1	1	1	1	1
0	1	0	1	0	0	0	1	0	0
0	1	1	1	0	1	1	1	1	1
1	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	1	0	0	0
1	1	0	0	1	0	1	1	1	1
1	1	1	0	1	0	1	1	1	1

Proof of theorem 13a by using previous theorems.

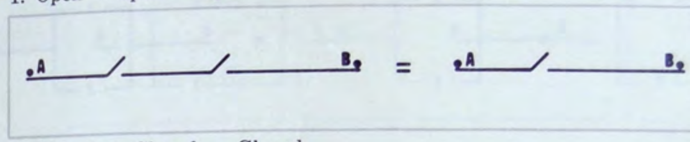
$$(X+Y)(\overline{X}+Z) = (X+Y)\overline{X} + (X+Y)Z \text{ by theorem 3a}$$
$$= \overline{X}(X+Y) + Z(X+Y) \text{ by theorem 1b}$$
$$= (\overline{X}\cdot X + \overline{X}\cdot Y) + Z(X+Y) \text{ by theorem 3a}$$
$$= (X\cdot \overline{X} + \overline{X}\cdot Y) + Z(X+Y) \text{ by theorem 1b}$$
$$= (0 + \overline{X}\cdot Y) + Z(X+Y) \text{ by theorem 8b}$$
$$= (\overline{X}\cdot Y) + Z(X+Y) \text{ by theorem 9a}$$
$$= \overline{X}\cdot Y + Z(X+\overline{X}Y) \text{ by theorem 11b}$$
$$= \overline{X}\,Y + (Z\cdot X + Z(\overline{X}\,Y)) \text{ by theorem 3b}$$
$$= \overline{X}\,Y + (Z(\overline{X}\,Y) + ZX) \text{ by theorem 1a}$$
$$= (\overline{X}\,Y + Z(\overline{X}\,Y)) + Z\,X \text{ by theorem 2a}$$
$$= (1\,(\overline{X}\,Y) + Z(\overline{X}\,Y)) + Z\,X \text{ by theorem 9b}$$
$$= ((\overline{X}\,Y)\cdot 1 + (\overline{X}\,Y)Z) + X\,Z \text{ by theorem 1b}$$
$$= (\overline{X}\,Y)(1+Z) + X\,Z \text{ by theorem 3a}$$
$$= (\overline{X}\,Y)\cdot 1 + X\,Z \text{ by theorem 10a}$$
$$= X\,Z + (\overline{X}\,Y)\cdot 1 \text{ by theorem 1a}$$
$$= X\,Z + 1\cdot(\overline{X}\,Y) \text{ by theorem 1b}$$
$$= X\,Z + \overline{X}\,Y \text{ by theorem 9b}$$

We will now place a physical interpretation on our abstract mathematical system. Suppose we say that X, Y, Z, W, etc. are electrical switches. Suppose we interpret the "O" above as the switch being open and the "1" above as the switch being closed, or on. Now we have to interpret the + and the • in our system. By X+Y, or switch 1 + switch 2, we mean that the switches are in parallel. By X • Y, or switch 1 • switch 2, we mean that the switches are in series. Hence we have:

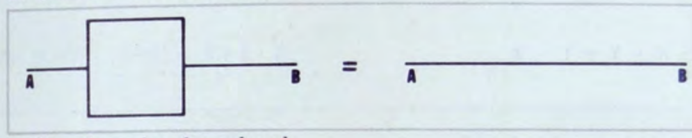


We will now check to see if our postulates are true with this interpretation.

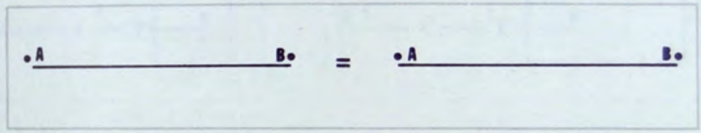
1. Open • open = open



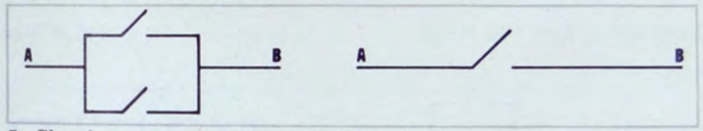
2. Closed + Closed = Closed



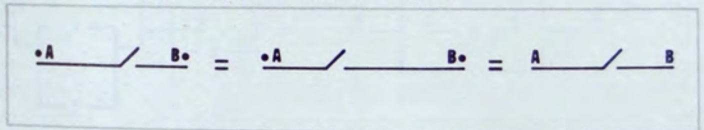
3. Closed • closed = closed



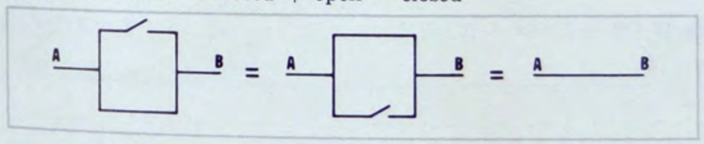
4. Open + open = open



5. Closed • open = open • closed = open

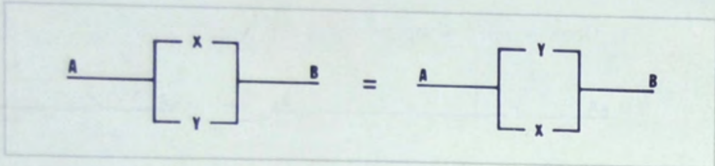


6. Open + closed = closed + open = closed

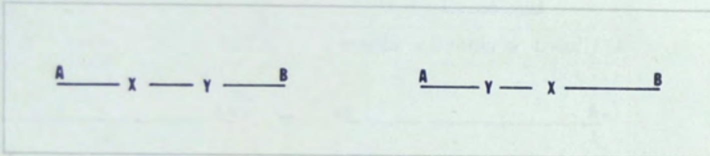


From this we see that all of the postulates are true. Hence, our theorems are true. We will now write down the theorems and under each we will put the equivalent statement in this physical system.

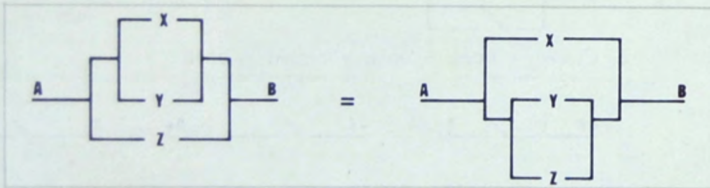
Theorem 1a: X + Y = Y + X



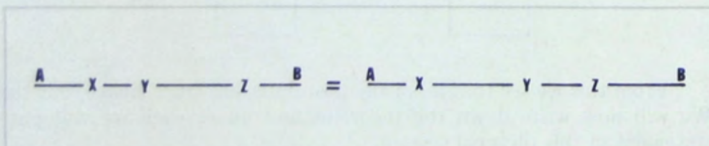
Theorem 1b: X · Y = Y · X



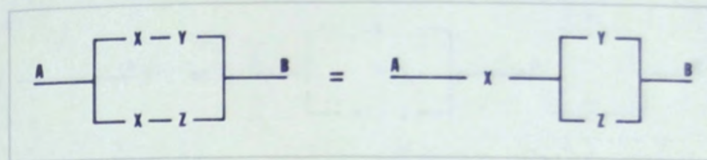
Theorem 2a: (X+Y) + Z = X + (Y+Z)



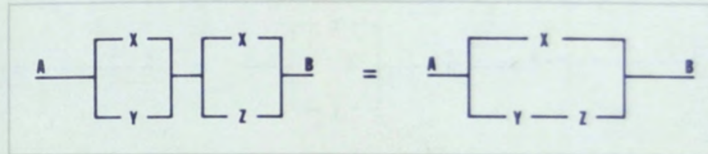
Theorem 2b: (X · Y) · Z = X · (Y · Z)



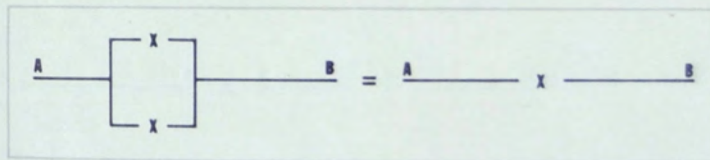
Theorem 3a: X · Y + X · Z = X · (Y+Z)



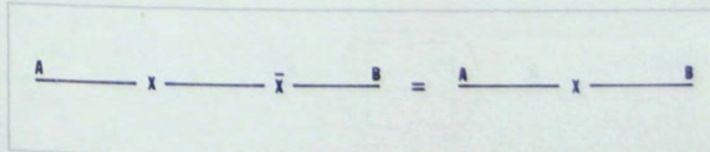
Theorem 3b: (X+Y) · (X+Z) = X+Y · Z



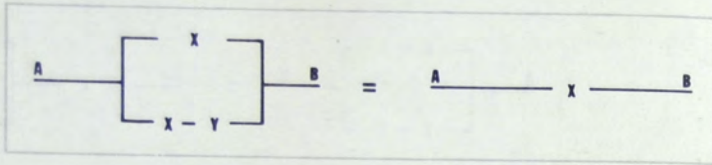
Theorem 4a: X + X = X



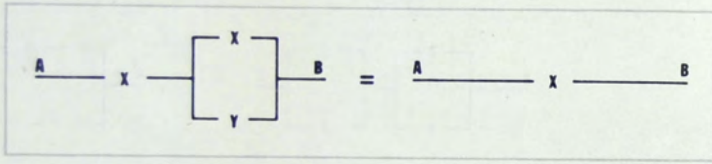
Theorem 4b: X · X = X



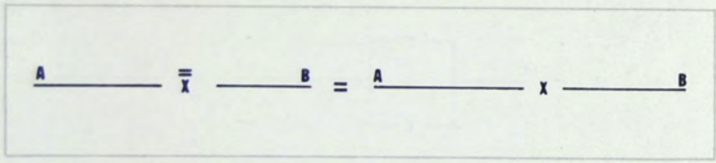
Theorem 5a: $X + X \cdot Y = X$.



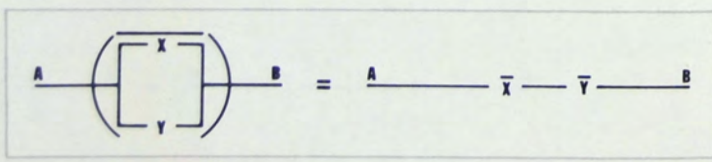
Theorem 5b: $X \cdot (X + Y) = X$.



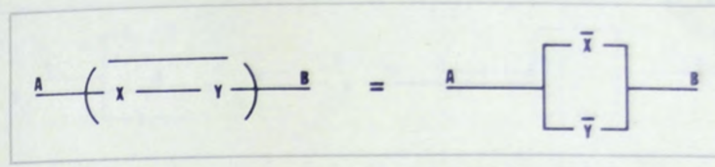
Theorem 6: $\overline{\overline{(X)}} = X$.



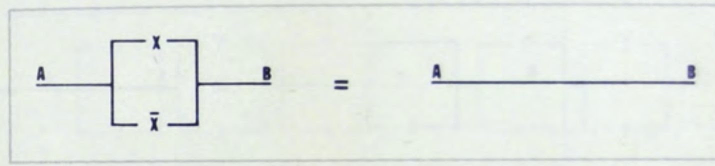
Theorem 7a: $\overline{(X + Y)} = \overline{X} \cdot \overline{Y}$



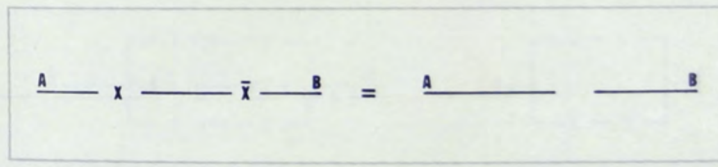
Theorem 7b: $\overline{(X \cdot Y)} = \overline{X} + \overline{Y}$



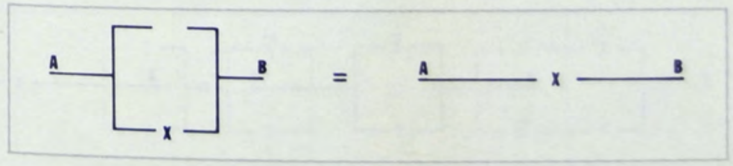
Theorem 8a: $X + \overline{X} = 1$



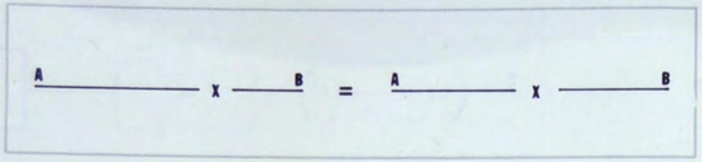
Theorem 8b: $X \cdot \overline{X} = 0$



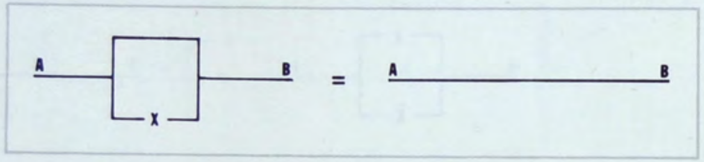
Theorem 9a: $0 + X = X$



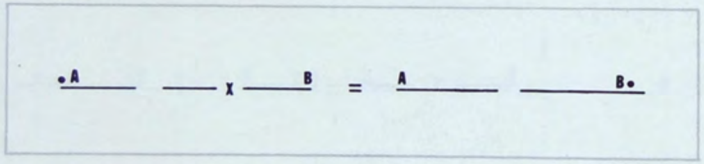
Theorem 9b: $1 \cdot X = X$



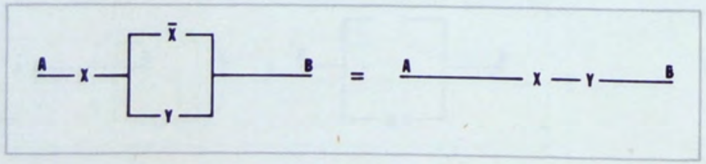
Theorem 10a: $1 + X = 1$



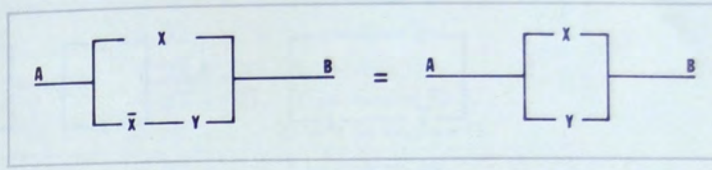
Theorem 10b: $0 \cdot X = 0$



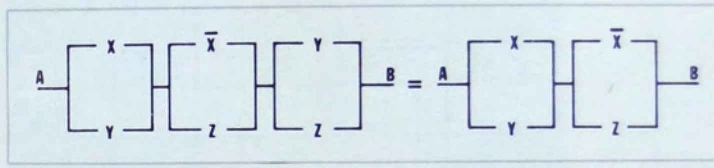
Theorem 11a: $X(\overline{X} + Y) = X \cdot Y$



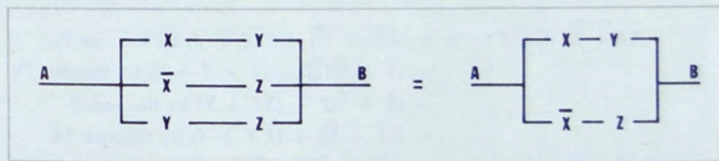
Theorem 11b: $X + \overline{X} \cdot Y = X + Y$



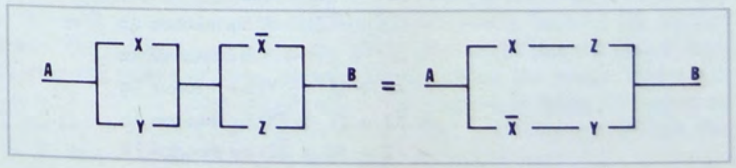
Theorem 12a: $(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)$



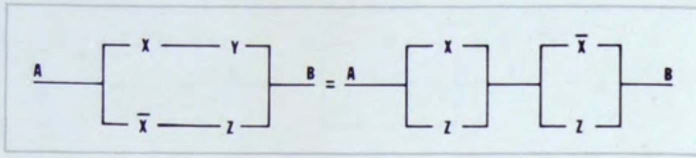
Theorem 12b: $X \cdot Y + \overline{X} \cdot Z + Y \cdot Z = X \cdot Y + \overline{X} \cdot Z$



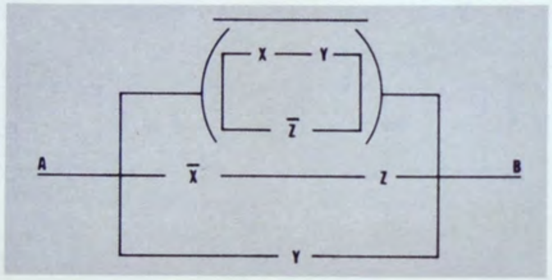
Theorem 13a: $(X + Y) \cdot (\overline{X} + Z) = X \cdot Z + \overline{X} \cdot Y$



Theorem 13b: $X \cdot Y + \overline{X} \cdot Z = (X + Z) \cdot (X + Y)$



Now, suppose that we have a complicated network of switches, we would like to simplify our circuit.



We would first write out the function for the circuit. Hence:

$\overline{XY + \overline{Z}} + (\overline{X}Z + Y) = (\overline{XY}) \cdot (\overline{\overline{Z}}) + (\overline{X}Z + Y)$ by theorem 7a

$= (\overline{X} + \overline{Y})(\overline{\overline{Z}}) + (\overline{X} \cdot Z + Y)$ by theorem 7b

$= (\overline{X} + \overline{Y})Z + (\overline{X}Z + Y)$ by theorem 6

$= Z(\overline{X} + \overline{Y}) + (\overline{X}\,Z + Y)$ by theorem 1b

$= (Z\overline{X} + Z\overline{Y}) + (\overline{X}\,Z + Y)$ by theorem 3a

$= ((Z\overline{X} + Z\overline{Y}) + \overline{X}\,Z) + Y$ by theorem 2a

$= ((Z\overline{Y} + Z\overline{X}) + \overline{X}Z) + Y$ by theorem 1a

$= (Z\overline{Y} + (Z\overline{X} + \overline{X}Z)) + Y$ by theorem 2a

$= (Z\overline{Y} + (\overline{X}Z + \overline{X}Z)) + Y$ by theorem 1b

$= (Z\overline{Y} + \overline{X}Z) + Y$ by theorem 8a

$= (\overline{X}Z + Z\overline{Y}) + Y$ by theorem 1a

$= \overline{X}Z + (Z\overline{Y} + Y)$ by theorem 2a

$= \overline{X}Z + (Y + Z\overline{Y})$ by theorem 1a

$= \overline{X}Z + (Y + \overline{Y}Z)$ by theorem 1b

$= \overline{X}Z + (Y + Z)$ by theorem 11b

$= \overline{X}Z + (Z + Y)$ by theorem 1a

$= (\overline{X}Z + Z) + Y$ by theorem 2a

$= (Z\overline{X} + Z)\ X\ Y$ by theorem 1b

$= (Z\overline{X} + 1Z) + Y$ by theorem 9b

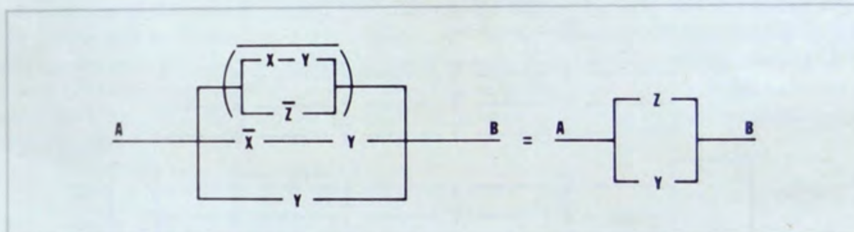
$= (Z\overline{X} + Z \cdot 1) + Y$ by theorem 1b

$= Z(\overline{X} + 1) + Y$ by theorem 3a

$= Z \cdot 1 + Y$ by theorem 10a

$= Z + Y$ by theorem 9b

Hence we have:



Once upon a time a father had a beautiful daughter named "X". The father, the daughter and the mother, named "Y", were spending the summer on a small island camp. Two of "X" 's suitors, "W" and "Z", accompanied them. "X" 's father is very fond of fishing. However, he does not like for "X" to be with either "W" or "Z", nor with both of them, unless "Y" is present. He finds that it requires some thought, when he looks up from his boat and identifies the figures on the beach, to deduce whether or not "X" is in the house or on the beach with one or both of the suitors without her mother. The problem is to design a "black box" such that the father may close switches bearing the names of those persons he can see on the beach, and a red light will come on any time "X" and either of her boy friends, or both, are either in the house or on the beach without the mother being present. Hence we see that the father has trouble if any one of the following shows up on the beach:

$$XZW,\ XZ,\ XW,\ YZ,\ YW,\ Y$$

Now remembering our notation, we may write:

$X\overline{Y} \cdot ZW$

$X\overline{Y} \cdot Z\overline{W}$

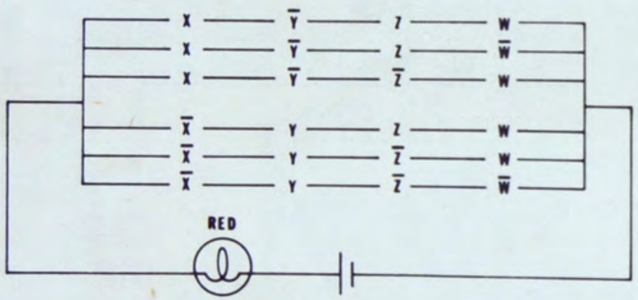
$X\overline{Y} \cdot \overline{Z} \cdot W$

$\overline{X}\ YZ\overline{W}$

$\overline{X}\ Y\overline{Z}\ W$

$\overline{X}\ Y\overline{Z}\ \overline{W}$

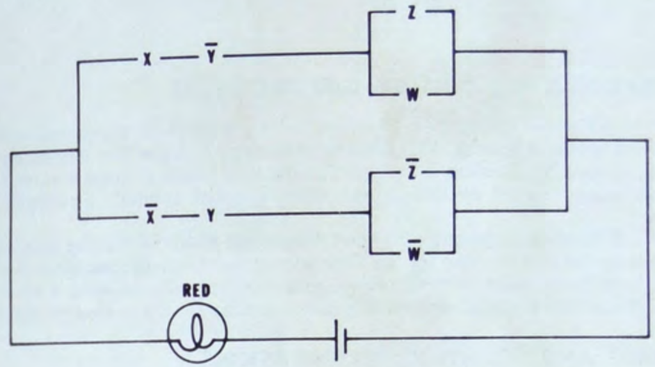
Hence, the following circuit would serve our purpose:



We would like to simplify this circuit if possible. Hence, in our algebra, we have:

$$X\overline{Y}\ ZW + X\overline{Y}\ Z\overline{W} + X\overline{Y}\ \overline{Z}\ W + \overline{X}\ YZ\overline{W} + \overline{X}\ Y\overline{Z}\ W + \overline{X}\ Y\overline{Z}\ \overline{W}$$
$$= X\overline{Y}\ Z\ (W + \overline{W}) + X\overline{Y}\ \overline{Z}\ W + \overline{X}\ YZ\overline{W} + \overline{X}\ Y\overline{Z}\ (W + \overline{W})$$
$$= X\overline{Y}\ Z + X\overline{Y}\ \overline{Z}\ W + \overline{X}\ YZ\overline{W} + \overline{X}\ Y\overline{Z}$$
$$= X\overline{Y}\ (Z + \overline{Z}\ W) + \overline{X}Y\ (Z\overline{W} + \overline{Z})$$
$$X\overline{Y}\ (Z + W) + \overline{X}Y(\overline{Z} + \overline{W})$$

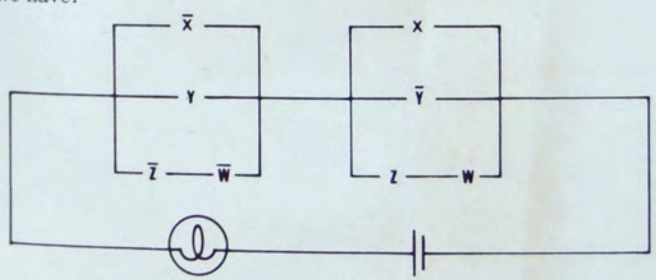
Hence we have:



This will be the circuit that will be built into the father's black box. Everything was fine until one of the suitors slipped in a burned out bulb. The next day the father was out fishing and he turned on the switches and nothing happened. Everything is all right, he thought. But then he looked up and saw only Y and W. Something is wrong! He decided that he must have a box that would burn green if everything is all right and red if there is trouble. How can he do this? We have only to build in the complimentary circuit. Hence:

$$\overline{(X\overline{Y}(Z + W) + \overline{X}Y(Z + \overline{W}))} =$$
$$\overline{(X\overline{Y}(Z + W))} \cdot \overline{(\overline{X}Y(\overline{Z} + \overline{W}))} =$$
$$(\overline{X} + \overline{\overline{Y}}) + \overline{(Z + W)}) \cdot ((\overline{\overline{X}}) + \overline{Y} + \overline{(\overline{Z} + \overline{W})}\ ) =$$
$$(\overline{X} + Y + \overline{Z}\overline{W}) \cdot (X + \overline{Y} + ZW)$$

Hence, we have:



By building the above circuits in the same box, he has his problem solved.

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